

# Nuclear thermodynamics and the in-medium chiral condensate

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## Abstract

The temperature dependence of the chiral condensate in isospin-symmetric nuclear matter at varying baryon density is investigated using thermal in-medium chiral effective field theory. This framework provides a realistic approach to the thermodynamics of the correlated nuclear many-body system and permits calculating systematically the pion-mass dependence of the free energy per particle. One- and two-pion exchange processes,  $\Delta(1232)$ -isobar excitations, Pauli blocking corrections and three-body correlations are treated up to and including three loops in the expansion of the free energy density. It is found that nuclear matter remains in the Nambu-Goldstone phase with spontaneously broken chiral symmetry in the temperature range  $T \lesssim 100$  MeV and at baryon densities at least up to about twice the density of normal nuclear matter,  $2\rho_0 \simeq 0.3 \text{ fm}^{-3}$ . Effects of the nuclear liquid-gas phase transition on the chiral condensate at low temperatures are also discussed.

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The chiral condensate  $\langle \bar{q}q \rangle$ , i.e. the expectation value of the scalar quark density, plays a fundamental role as an order parameter of spontaneously broken chiral symmetry in the hadronic low-energy phase of QCD. The variation of  $\langle \bar{q}q \rangle$  with temperature and baryon density is a key issue for locating the chiral transition boundary in the QCD phase diagram. The melting of the condensate at high temperatures and/or densities determines the crossover from the Nambu-Goldstone phase to the Wigner-Weyl realization of chiral symmetry in QCD.

It is thus of principal interest to perform a systematically organized calculation of the thermodynamics of the chiral condensate. Such a calculation requires knowledge of the dependence of the free energy density on the light quark mass (or equivalently, on the pion mass). The appropriate framework for such a task is in-medium chiral effective field theory with its explicit access to one- and two-pion exchange dynamics and resulting two- and three-body correlations in the presence of a nuclear medium.

Previous studies of the in-medium variation of the chiral condensate were mostly concerned with the density dependence of  $\langle \bar{q}q \rangle$  at zero temperature, using different approaches such as QCD sum rules [1] or models [2,3] based on the boson exchange

phenomenology of nuclear forces. Temperature effects have been included in schematic Nambu-Jona-Lasinio (NJL) approaches [4,5]. Such NJL models work with quarks as quasiparticles and provide useful insights into dynamical mechanisms behind spontaneous chiral symmetry breaking and restoration, but they do not properly account for nucleons and their correlations, a prerequisite for a more realistic treatment.

The present work extends a previous chiral effective field theory calculation [6] of the density-dependent in-medium condensate  $\langle \bar{q}q \rangle(\rho)$  to finite temperatures  $T$ . Corrections to the linear density approximation are obtained by differentiating the interaction parts of the free energy density of isospin-symmetric nuclear matter with respect to the (squared) pion mass. Effects from one-pion exchange (with  $m_\pi$ -dependent vertex corrections), iterated  $1\pi$ -exchange, and irreducible  $2\pi$ -exchange including intermediate  $\Delta(1232)$ -isobar excitations, with Pauli-blocking corrections are systematically treated up to three-loop order. The dominant nuclear matter effects on the dropping condensate are supplemented by a further small reduction due to interacting thermal pions. To anticipate the result: we find that the delayed tendency towards chiral symmetry restoration with increasing baryon density  $\rho$ , observed at  $T = 0$  [6] in the same framework, gets gradually softened with increasing temperature. An approximately linear decrease of the quark condensate with increasing  $\rho$  is recovered at temperatures around  $T \simeq 100$  MeV. However, no rapid drive towards a first order chiral phase transition is seen, at least up to  $\rho \lesssim 2\rho_0$  where  $\rho_0 = 0.16 \text{ fm}^{-3}$  is the density of normal nuclear matter. The only phase transition known for nuclear matter in this density range is the first order transition between an interacting Fermi gas and a Fermi liquid, with its broad coexistence region extending from low densities up to about  $\rho_0$ . Signatures of this liquid-gas transition are nonetheless visible also in the chiral condensate at low temperatures and will be discussed in this work.

Our starting point is the free energy density,  $\mathcal{F}(\rho, T) = \rho \bar{F}(\rho, T)$ , of isospin-symmetric spin-saturated nuclear matter, with  $\bar{F}(\rho, T)$  the free energy per particle. In the approach to nuclear matter based on in-medium chiral perturbation theory [7–9] the free energy density is given by a sum of convolution integrals of the form,

$$\begin{aligned} \rho \bar{F}(\rho, T) = & 4 \int_0^\infty dp p \mathcal{K}_1 n(p) + \int_0^\infty dp_1 \int_0^\infty dp_2 \mathcal{K}_2 n(p_1) n(p_2) \\ & + \int_0^\infty dp_1 \int_0^\infty dp_2 \int_0^\infty dp_3 \mathcal{K}_3 n(p_1) n(p_2) n(p_3) + \rho \bar{\mathcal{A}}(\rho, T), \end{aligned} \quad (1)$$

where  $\mathcal{K}_1, \mathcal{K}_2$  and  $\mathcal{K}_3$  are one-body, two-body and three-body kernels, respectively. The last term, the so-called anomalous contribution  $\bar{\mathcal{A}}(\rho, T)$  is a special feature at finite temperatures [10] with no counterpart in the calculation of the ground state energy density at  $T = 0$ . As shown in ref.[7] the anomalous contribution arising in the present context from second-order pion exchange has actually very little influence on the equation of state of nuclear matter at moderate temperatures  $T < 50$  MeV.

The quantity

$$n(p) = \frac{p}{2\pi^2} \left[ 1 + \exp \frac{p^2/2M_N - \tilde{\mu}}{T} \right]^{-1} \quad (2)$$

denotes the density of nucleon states in momentum space. It is the product of the temperature dependent Fermi-Dirac distribution and a kinematical prefactor  $p/2\pi^2$  which has been included in  $n(p)$  for convenience.  $M_N$  stands for the (free) nucleon mass. The particle density  $\rho$  is calculated as

$$\rho = 4 \int_0^\infty dp p n(p). \quad (3)$$

This relation determines the dependence of the effective one-body chemical potential  $\tilde{\mu}(\rho, T; M_N)$  on the thermodynamical variables  $(\rho, T)$  and indirectly also on the nucleon mass  $M_N$ . The one-body kernel  $\mathcal{K}_1$  in eq.(1) provides the contribution of the non-interacting nucleon gas to the free energy density and it reads [7]:

$$\mathcal{K}_1(p) = M_N + \tilde{\mu} - \frac{p^2}{3M_N} - \frac{p^4}{8M_N^3}. \quad (4)$$

The first term in  $\mathcal{K}_1$  gives the leading contribution (density times nucleon rest mass) to the free energy density. The remaining terms account for (relativistically improved) kinetic energy corrections.

The two- and three-body kernels,  $\mathcal{K}_2$  and  $\mathcal{K}_3$ , specifying all one- and two-pion exchange processes up to three loop order for the free energy density, have already been given in explicit form in refs.[7–9] and will not be repeated here. We recall from our earlier works that after fixing only a few contact terms the free energy density computed from these interaction kernels provides a realistic nuclear equation of state up to densities  $\rho \lesssim 2\rho_0$ . What matters in the following will be the dependence of the kernels  $\mathcal{K}_2$  and  $\mathcal{K}_3$  on the light quark mass,  $m_q$ , or equivalently, on the pion mass,  $m_\pi$ , that is introduced by pion propagators and by pion loops. In-medium chiral effective field theory is the appropriate framework to quantify this pion-mass dependence in a systematic and reliable way.

Application of the Feynman-Hellmann theorem establishes an exact connection between the temperature and density dependent in-medium quark condensate  $\langle \bar{q}q \rangle(\rho, T)$  and the derivative of the free energy density of (isospin-symmetric, spin-saturated) nuclear matter with respect to the light quark mass  $m_q$ . Using the Gell-Mann-Oakes-Renner relation  $m_\pi^2 f_\pi^2 = -m_q \langle 0 | \bar{q}q | 0 \rangle$ , one finds for the ratio of the in-medium to vacuum quark condensate

$$\frac{\langle \bar{q}q \rangle(\rho, T)}{\langle 0 | \bar{q}q | 0 \rangle} = 1 - \frac{\rho}{f_\pi^2} \frac{\partial \bar{F}(\rho, T)}{\partial m_\pi^2}, \quad (5)$$

where the derivative with respect to  $m_\pi^2$  is to be taken at fixed  $\rho$  and  $T$ . The quantities  $\langle 0 | \bar{q}q | 0 \rangle$  (vacuum quark condensate) and  $f_\pi$  (pion decay constant) are to be understood as taken in the chiral limit,  $m_q \rightarrow 0$ . Likewise,  $m_\pi^2$  stands for the leading linear term in the quark mass expansion of the squared pion mass.

In the one-body kernel  $\mathcal{K}_1$  the quark (or pion) mass dependence is implicit via its dependence on the nucleon mass  $M_N$ . The condition  $\partial \rho / \partial M_N = 0$  applied to eq.(3) leads to the following dependence of the effective one-body chemical potential  $\tilde{\mu}$  on the nucleon mass  $M_N$ :

$$\frac{\partial \tilde{\mu}}{\partial M_N} = \frac{3\rho}{2M_N\Omega_0''}, \quad \Omega_0'' = -4M_N \int_0^\infty dp \frac{n(p)}{p}. \quad (6)$$

The nucleon sigma term  $\sigma_N = \langle N | m_q \bar{q}q | N \rangle = m_\pi^2 \partial M_N / \partial m_\pi^2$  measures the variation of the nucleon mass with the quark (or pion) mass. Combining both relationships leads to the following  $m_\pi^2$ -derivative of the one-body kernel:

$$\frac{\partial \mathcal{K}_1}{\partial m_\pi^2} = \frac{\sigma_N}{m_\pi^2} \left\{ 1 + \frac{3\rho}{2M_N\Omega_0''} + \frac{p^2}{3M_N^2} + \frac{3p^4}{8M_N^4} \right\}. \quad (7)$$

In the limit of zero temperature,  $T = 0$ , the terms in eq.(7) reproduce the linear decrease of the chiral condensate with density. The kinetic energy corrections account for the (small) difference between the scalar and the vector (i.e. baryon number) density. In the actual calculation we use the chiral expansion of the nucleon sigma term  $\sigma_N$  to order  $\mathcal{O}(m_\pi^4)$  as given in eq.(19) of ref.[6]. The empirical value of the nucleon sigma term (at the physical pion mass  $m_\pi = 135$  MeV) is  $\sigma_N = (45 \pm 8)$  MeV [11]. Recent results for the quark mass dependence of baryon masses from lattice QCD and accurate chiral extrapolations [12] tend to give smaller values of this sigma term, but still consistent with the empirical  $\sigma_N$  within errors. We use the central value  $\sigma_N = 45$  MeV in the present calculations, keeping in mind however that the existing error band in the determination of this quantity is still a primary source of uncertainties in following discussion.

For the two- and three-body kernels,  $\mathcal{K}_2$  and  $\mathcal{K}_3$ , related to one-pion exchange and iterated one-pion exchange, explicit expressions have been given in ref.[7]. Their derivatives with respect to the squared pion mass,  $\partial \mathcal{K}_{2,3} / \partial m_\pi^2$ , are hence obvious and do not need to be written out here. The same applies to the anomalous contribution  $\bar{\mathcal{A}}(\rho, T)$  (see eqs.(14,15) in ref.[7]) and to the two- and three-body kernels related to  $2\pi$ -exchange with excitation of virtual  $\Delta(1232)$ -isobars (see section 6 in ref.[8]). In case of the one-pion exchange contribution we include the  $m_\pi$ -dependent vertex correction factor  $\Gamma(m_\pi)$  as discussed in section 2.1 of ref.[6]. The short-distance contact term that produces a  $T$ -independent correction of order  $\rho^2$  to the in-medium condensate is treated exactly in the same way as in ref.[6], i.e. all terms with a non-analytical quark-mass dependence generated by pion-loops are taken into account.

Let us now turn to some three-loop contributions that are new and of special relevance for the in-medium chiral condensate. The first one comes from the pion self-energy diagram shown in Fig.1. It gives rise to the following  $m_\pi^2$ -derivative of the two-body kernel:

$$\begin{aligned} \frac{\partial \mathcal{K}_2^{(\pi)}}{\partial m_\pi^2} = & \frac{3g_A^2 m_\pi^2}{\pi^2 (4f_\pi)^4} \left\{ \bar{\ell}_3 [(X_{12}^-)^2 - (X_{12}^+)^2] \right. \\ & \left. + (4\bar{\ell}_3 - 1) (X_{12}^+ - X_{12}^-) + (2\bar{\ell}_3 - 1) \ln \frac{X_{12}^-}{X_{12}^+} \right\}, \end{aligned} \quad (8)$$

with the abbreviations  $X_{ij}^\pm = [1 + (p_i \pm p_j)^2 / m_\pi^2]^{-1}$  and the  $\pi\pi$  low-energy constant  $\bar{\ell}_3 \simeq 3$ .

The chiral  $\pi\pi NN$  contact vertex proportional to  $c_1 m_\pi^2$  generates  $2\pi$ -exchange Hartree and Fock diagrams, also shown in Fig.1. Concerning the free energy density  $\rho \bar{F}(\rho, T)$  or the equation of state of nuclear matter their contributions are actually almost negligible.

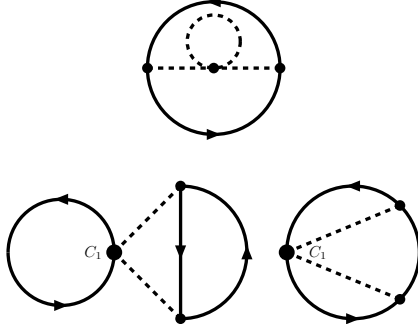


Fig. 1. Three-loop contributions to the free energy density of nuclear matter that are relevant for the in-medium chiral condensate. Upper diagram: pion self-energy correction; lower diagrams: two-pion exchange Hartree and Fock terms involving the  $\pi\pi NN$  contact interaction proportional to the low-energy constant  $c_1$ .

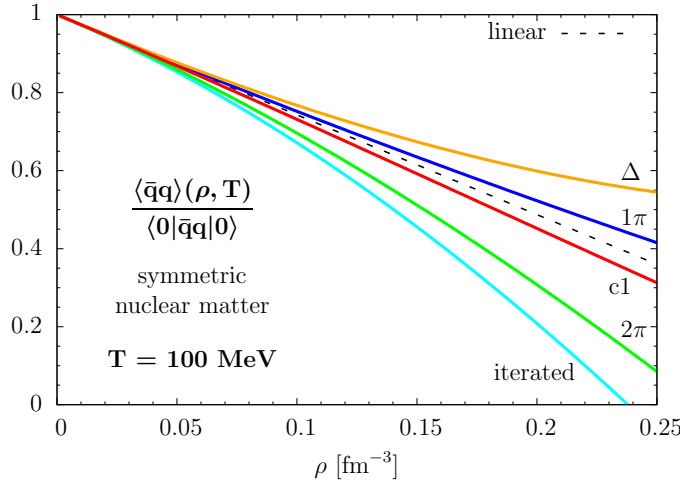


Fig. 2. Density dependence of the chiral condensate in isospin-symmetric nuclear matter at temperature  $T = 100$  MeV. Starting from the linear density dependence (dashed curve) characteristic of the free nucleon Fermi gas, the following interaction contributions are successively added: one-pion exchange Fock term ( $1\pi$ ), second order (iterated) pion exchange, irreducible two-pion exchange ( $2\pi$ ), two- and three-body contributions from  $2\pi$  exchange with intermediate  $\Delta$  excitations ( $\Delta$ ), and two-pion exchange with  $\pi\pi NN$  vertex involving the low-energy constant  $c_1$ . Pauli blocking effects are included throughout.

However, when taking the derivative with respect to  $m_\pi^2$  as required for the calculation of the in-medium condensate, these contributions turn out to be of similar importance as other interaction terms. The corresponding contribution to the derivative of the two-body kernel reads:

$$\frac{\partial \mathcal{K}_2^{(c_1)}}{\partial m_\pi^2} = \frac{g_A^2 c_1 m_\pi^3}{8\pi f_\pi^4} \left\{ G\left(\frac{p_1 + p_2}{2m_\pi}\right) - G\left(\frac{p_1 - p_2}{2m_\pi}\right) \right\}, \quad (9)$$

with the auxiliary function:

$$G(x) = 8x(3 + x^2) \arctan x - 5 \ln(1 + x^2) - 100x^2. \quad (10)$$

The  $2\pi$ -exchange Hartree diagram with one  $c_1 m_\pi^2$ -vertex contributes the following piece to the three-body kernel:

$$\frac{\partial \mathcal{K}_3^{(c_1, H)}}{\partial m_\pi^2} = \frac{6g_A^2 c_1 p_3}{f_\pi^4} \left\{ (X_{12}^+ - X_{12}^-)(X_{12}^+ + X_{12}^- - 3) + \ln \frac{X_{12}^+}{X_{12}^-} \right\}, \quad (11)$$

while the three-body term associated with the  $2\pi$ -exchange Fock diagram with one  $c_1 m_\pi^2$ -vertex gives:

$$\begin{aligned} \frac{\partial \mathcal{K}_3^{(c_1)}}{\partial m_\pi^2} &= \frac{3g_A^2 c_1}{f_\pi^4} \left[ \frac{p_2}{p_3} + \frac{p_3^2 - p_2^2 - m_\pi^2}{4p_3^2} \ln \frac{X_{23}^-}{X_{23}^+} \right] \\ &\times \left[ p_1 + \frac{p_3^2 - p_1^2 - 3m_\pi^2}{4p_3} \ln \frac{X_{13}^-}{X_{13}^+} + (p_1 + p_3)X_{13}^+ + (p_1 - p_3)X_{13}^- \right]. \end{aligned} \quad (12)$$

Last not least we incorporate the effects of thermal pions. Through its  $m_\pi^2$ -derivative the pressure (or free energy density) of thermal pions gives rise to a further reduction of the  $T$ -dependent in-medium condensate. In the two-loop approximation of chiral perturbation theory including effects from the  $\pi\pi$ -interaction one finds the following shift of the condensate ratio in the presence of the pionic heat bath [13–15]:

$$\frac{\delta \langle \bar{q}q \rangle(T)}{\langle 0 | \bar{q}q | 0 \rangle} = - \frac{3m_\pi^2}{(2\pi f_\pi)^2} H_3\left(\frac{m_\pi}{T}\right) \left\{ 1 + \frac{m_\pi^2}{8\pi^2 f_\pi^2} \left[ H_3\left(\frac{m_\pi}{T}\right) - H_1\left(\frac{m_\pi}{T}\right) + \frac{2 - 3\bar{\ell}_3}{8} \right] \right\}, \quad (13)$$

with the functions  $H_{1,3}(m_\pi/T)$  defined by integrals over the Bose distribution of thermal pions:

$$H_1(y) = \int_y^\infty dx \frac{1}{\sqrt{x^2 - y^2}(e^x - 1)}, \quad H_3(y) = y^{-2} \int_y^\infty dx \frac{\sqrt{x^2 - y^2}}{e^x - 1}. \quad (14)$$

We proceed with a presentation of results. As input we consistently use the same parameters in the chiral limit as in our previous works [6], namely:  $f_\pi = 86.5$  MeV,  $g_A = 1.224$ ,  $c_1 = -0.93$  GeV $^{-1}$  and  $M_N = 882$  MeV. Concerning the contact term representing unresolved short-distance dynamics, we recall from ref. [6] that its quark mass dependence, estimated from recent lattice QCD results [16], is negligibly small compared to that of the intermediate and long range (pion-exchange driven) pieces.

It is worth pointing out again that in-medium chiral perturbation theory with this input produces a realistic nuclear equation of state [8,9], including a proper description of the thermodynamics of the liquid-gas phase transition. Apart from temperature  $T$ , the additional “small” parameter in this approach is the nucleon Fermi momentum  $p_F$  in comparison with the chiral scale,  $4\pi f_\pi \sim 1$  GeV. Our three-loop calculation of the free energy density is reliable up to about twice the density of normal nuclear matter. It can be trusted over a temperature range (up to  $T \sim 100$  MeV) in which the hot and dense matter still remains well inside the phase of spontaneously broken chiral symmetry.

Fig.2 shows a representative example, at  $T = 100$  MeV, displaying stepwise the effects of interaction contributions to the density dependence of  $\langle \bar{q}q \rangle(\rho, T)$  arising from the

$m_\pi^2$ -derivative of the chiral two- and three-body kernels  $\mathcal{K}_{2,3}$ . As in the  $T = 0$  case studied previously [6], the pion-mass dependence of correlations involving virtual  $\Delta(1232)$  excitations turns out to be specifically important in delaying the tendency towards chiral symmetry restoration as the density increases. Once all one- and two-pion exchange processes contributing to  $\partial\mathcal{K}_2/\partial m_\pi^2$  and  $\partial\mathcal{K}_3/\partial m_\pi^2$  are added up, the chiral condensate at  $T = 100$  MeV recovers the linear density dependence characteristic of a free Fermi gas. However, this recovery is the result of a subtle balance between attractive and repulsive correlations and their detailed pion-mass dependences. Had we taken into account only iterated one-pion and irreducible two-pion exchanges, the system would have become unstable not far above normal nuclear matter density as can be seen in Fig.2. In fact, this instability would have appeared at even much lower densities in the chiral limit ( $m_\pi \rightarrow 0$ ). This emphasizes once more not only the importance of terms including  $\Delta(1232)$ -excitations, but also the significance of explicit chiral symmetry breaking by small but non-zero quark masses in QCD and the resulting physical pion mass,  $m_\pi = 135$  MeV, in governing nuclear scales.

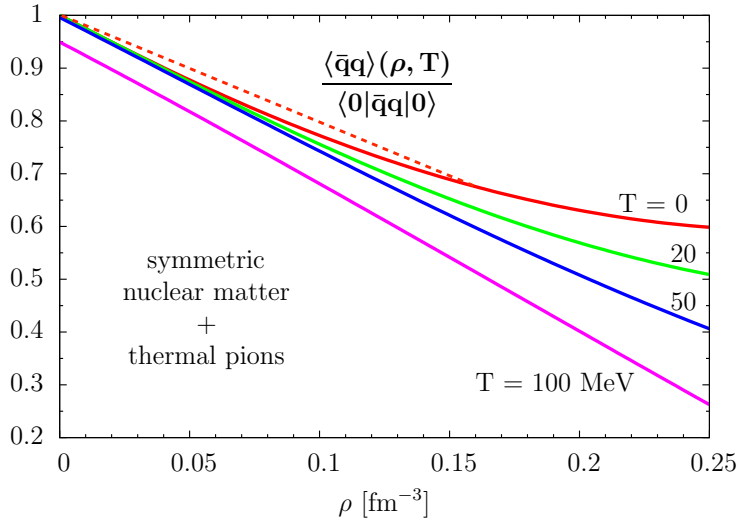


Fig. 3. Ratio of chiral condensate relative to its vacuum value as function of baryon density  $\rho$  in isospin-symmetric nuclear matter, for different temperatures up to  $T = 100$  MeV as indicated. The effects of thermal pions are included. The dashed line at  $T \simeq 0$  results through a Maxwell construction in the coexistence region of the nuclear liquid and gas phases.

Fig.3 shows the systematics in the variation of the chiral condensate with temperature  $T$  and baryon density  $\rho$ . These results include all nuclear correlation effects and also the (small) additional shift from thermal pions. The latter correction is visible only at the highest temperature considered here ( $T = 100$  MeV) where the chiral condensate at zero density starts to deviate from its vacuum value. The actual crossover transition at which the condensate drops continuously to zero is around  $T \sim 170$  MeV [17].

At zero temperature, the hindrance of the dropping condensate at densities beyond normal nuclear matter comes primarily from three-body correlations through  $\mathcal{K}_3$  which grow rapidly and faster than  $\mathcal{K}_2$  as the density increases. The heating of the system

reduces the influence of  $\mathcal{K}_3$  relative to  $\mathcal{K}_2$  continuously as the temperature rises, so that their balance at  $T = 100$  MeV produces a small net effect in comparison with the free Fermi gas.

At  $T = 0$ , the solid line in Fig.3 does not yet take into account the fact that the density range up to and including normal nuclear matter density covers the coexistence region of the nuclear liquid and gas phases [9]. Any first-order phase transition is expected to leave its mark also in other order parameters, and so it does for the chiral quark condensate. Based on the usual Maxwell construction, the dashed line in Fig.3 indicates this effect. It becomes much more pronounced when the chiral condensate at low temperatures is plotted as a function of the baryon chemical potential:

$$\mu = M_N + \left(1 + \rho \frac{\partial}{\partial \rho}\right) \bar{F}(\rho, T). \quad (15)$$

The discontinuity indicating the first-order liquid-gas transition at  $T$  smaller than the critical temperature for this transition,  $T_c \simeq 15$  MeV, is clearly visible in Fig. 4. At this point our results are consistent with a recent investigation aimed at an understanding of chemical freeze-out in heavy-ion collisions at large baryon densities [18], where a similar effect is found using a chiral meson-baryon model Lagrangian. Another side effect induced by the first-order liquid-gas phase transition in the discussion of the in-medium chiral condensate  $\langle \bar{q}q \rangle(\rho)$  is that the frequently advocated “low-density theorem” needs to be modified:

$$\frac{\langle \bar{q}q \rangle(\rho)}{\langle 0 | \bar{q}q | 0 \rangle} = 1 - \frac{\tilde{\sigma}_N}{m_\pi^2 f_\pi^2} \rho, \quad (16)$$

where the effective nucleon sigma term  $\tilde{\sigma}_N \simeq 36$  MeV measures the quark mass dependence of the sum  $M_N + \bar{E}_0$ , with  $\bar{E}_0 \simeq -16$  MeV the binding energy per particle of saturated nuclear matter. The usual version of eq.(16) with the nucleon sigma term  $\sigma_N$  in vacuum assumes that at sufficiently low densities nuclear matter could be treated as a non-interacting Fermi gas.

In summary, this is the first calculation of the quark condensate at finite temperature and density that systematically incorporates chiral two-pion exchange interactions in the nuclear medium. Correlations involving intermediate  $\Delta(1232)$ -isobar excitations (i.e. the strong spin-isospin polarizability of the nucleon) together with Pauli-blocking effects are demonstrated to play a crucial role in stabilizing the condensate at densities beyond that of the nuclear matter ground state. The results reported here set important nuclear physics constraints for the QCD equation of state at baryon densities and temperatures that are of interest e.g. in relativistic heavy-ion collisions. In particular, we find no indication of a first-order chiral phase transition at temperatures  $T \lesssim 100$  MeV and baryon densities at least up to about twice the density of normal nuclear matter.

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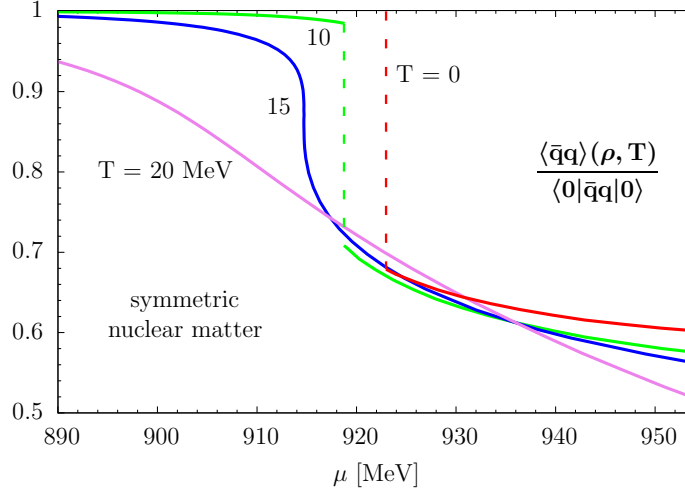


Fig. 4. Ratio of chiral condensate relative to its vacuum value as function of baryon chemical potential in symmetric nuclear matter at low temperatures characteristic of the nuclear liquid-gas phase coexistence region.

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